Substorm onset by plasma sheet divergence

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[1] It is necessary to understand current wedge formation in order to understand the cause of the substorm expansion phase. In the companion paper we used Geotail spacecraft observations to show that the cross-tail current reduction within the inner-plasma-sheet current wedge results from a process that leads to a reduction in equatorial plasma pressure and a substantial reduction in flux tube ion content. Here we use the single-species continuity equation for plasma sheet particles to identify a plausible cause of these plasma reductions and thus for current wedge formation and the initiation of the substorm expansion phase. Specifically, we find that a convection reduction, which follows a growth phase period of enhanced convection, should cause a divergence of plasma sheet particles driven by diamagnetic drift that leads to flux tube content reduction. We find that the reduction in flux tube content should be longitudinally localized to the premidnight to midnight region where the current wedge has been observed to initially form and that the reduction must initiate within the region of the equatorial mapping of the Harang discontinuity, consistent with ionospheric observations of substorm onset. We also find that the reduction in flux tube content should initially develop slowly and then develop more rapidly as the current wedge forms. This is consistent with observations which show that expansion phase aurora, and thus also the current wedge, develops slowly for a few minutes before brightening rapidly, and it is as required if plasma sheet divergence driven by diamagnetic drift leads to current wedge formation and initiation of the substorm expansion phase. *INDEX TERMS:* 2788 Magnetospheric Physics: Storms and substorms; 2764 Magnetospheric Physics: Plasma sheet; 2704 Magnetospheric Physics: Auroral phenomena (2407); 2431 Ionosphere: Ionosphere/magnetosphere interactions (2736); *KEYWORDS:* substorms, plasma sheet, geomagnetic tail, magnetosphere-ionosphere coupling, aurora

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1. Introduction

[2] One of the most important disturbances of the magnetosphere-ionosphere system is the expansion phase of substorms. It can be identified by active auroral displays that initiate within an ~2–3 hour local time region near the equatorward boundary of the auroral oval and then expand poleward and in local time [*Akasofu*, 1964]. Understanding substorm processes has been an active research topic for over 30 years. However, there is as yet no general agreement on what causes the substorm expansion.

[3] It is necessary to understand current wedge formation in order to understand the cause of the substorm expansion phase. In the companion paper [*Lyons et al.*, 2003b; hereafter referred to as Paper 1], Geotail measurements from within the equatorial inner plasma sheet at r ~ 10–13 R_E were used to show that a reduction in equatorial plasma pressure within the current wedge is a general feature of substorms. We also found that there is a significant reduction in flux tube ion content within the current wedge and that this reduction is sufficient to lead to the reduction in equatorial plasma pressure and cross-tail current of the current wedge. Here we offer an explanation, based on particle divergence driven by magnetic drift, for the reduction of plasma pressure and flux tube content that leads to current wedge formation and thus to the substorm expansion phase.

[4] It is generally accepted that substorm onset occurs after a ≳30 min growth-phase period of enhanced magnetospheric convection that enhances equatorial plasma pressure P_{eq} within the nightside plasma sheet. Pressure monotonically increase with decreasing equatorial radial distance r during the growth phase [*Kistler et al.*, 1992], reaching a peak near the inner edge of the plasma sheet, which is typically located at r ~ 5–7 R_E during the growth phase. The enhanced growth-phase pressure is associated with an enhanced cross-tail current that stretches the nightside magnetic field and reduces the component of magnetic field crossing the current sheet.

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[5] The expansion phase occurs in association with a region of reduced cross-tail current that is connected to the ionosphere by field-aligned currents. These field-aligned currents are directed out of (into) the ionosphere on the dusk (dawn) side of the region of cross-tail current reduction and close in the ionosphere via the westward electrojet that forms at expansion phase onset. The entire system consisting of the region of reduced cross-tail current, field-aligned currents, and the auroral electrojet is referred to as the "substorm current wedge" [e.g., *McPherron et al.*, 1973; *Moore et al.*, 1981]. The wedge currents enhance the component of magnetic field crossing the current sheet, thus dipolarizing the magnetic field within the inner plasma sheet. The substorm current wedge forms initially in the premidnight to midnight local-time sector of the inner plasma sheet [e.g., *Lopez et al.*, 1990; *Korth et al.*, 1991; *Lui et al.*, 1992; *Samson et al.*, 1992], but tailward of the peak in equatorial plasma sheet pressure [*Deehr and Lummerzheim*, 2001]. Electron acceleration by magnetic-field-aligned potential drops in the region of upward wedge currents forms the active auroral displays that occur during the substorm expansion phase. This auroral activity has been observed to initiate within the region of converging ionospheric electric fields that identifies the Harang discontinuity [*Nielson and Greenwald*, 1979; *Baumjohann et al.*, 1981; *Nielson*, 1991; *Hughes and Bristow*, 2003], implying that current wedge formation initiates along magnetic field lines that map to the region of the Harang discontinuity. After onset, the current wedge expands both westward and eastward [*Arnoldy and Moore*, 1983; *Nagai*, 1982, 1987] as well as radial outward [*Jacquey et al.*, 1991, 1993; *Ohtani et al.*, 1992].

[6] Observations have recently shown that the auroral enhancement that characterizes the substorm expansion phase first develops slowly, starting a few minutes prior to the time normally identified as substorm onset, and then develops more rapidly after onset [*Lyons et al.*, 2002; *Voronkov et al.*, 2003]. Since the expansion phase aurora is associated with the substorm current wedge, these auroral observations imply that the substorm current wedge also develops slowly for a few minutes prior to onset and then develops rapidly after onset. This behavior is like that of a classical instability, growing monotonically prior to expansion phase onset and growing more rapidly and becoming nonlinear after onset [*Voronkov et al.*, 2000]. Also, observations of dayside ionospheric convection have shown that the majority of substorms with well-defined onsets are associated with reductions in the strength of large-scale convection that is imparted to the magnetosphere from the solar wind, and that these reductions initiate a few minutes prior to expansion phase onset [*Lyons et al.*, 2003a, and references therein]. Since substorm aurora and the current wedge are nightside phenomena, they are not expected to modify dayside convection. Thus the above observations imply that growth of the substorm current wedge is initiated by a reduction in the strength of large-scale convection [*Lyons et al.*, 2003a].

[7] Understanding current wedge formation and thus the initiation of the substorm expansion phase requires understanding what causes the reduction in equatorial plasma pressure within the current wedge. However, how this reduction occurs presents an interesting puzzle. Adiabatic motion of an isotropic particle distribution within a flux tube conservers $T\mathcal{V}^{2/3}$, where T is temperature and $\mathcal{V} = \int ds/B$ is the flux tube volume per unit magnetic flux evaluated along field lines from the equator to the ionosphere. For no sources or losses of particles within a flux tube, the total number of particles $n\mathcal{V}$ within a flux tube is also conserved, where n is plasma density [e.g., *Wolf*, 1983; *Heinemann*, 1999]. Dipolarization of the magnetic field within the current wedge causes a decrease in $\mathcal{V}$ and would thus cause an increase in both n and T. This would lead to an increase in P_{eq}, which is inconsistent with the observations in Paper 1 and could not lead to current wedge formation. This suggests that the assumption of adiabatic motion without sources or losses may not be applicable to flux tubes during the substorm expansion phase.

[8] In this paper, we apply the continuity equation for plasma sheet particles derived by *Heinemann* [1999] to investigate the cause of the reduction in flux tube content. We find that a convection reduction following a growth-phase period of enhanced convection will cause a divergence of plasma sheet ions driven by diamagnetic drift that should lead to a reduction of flux tube content. We find that this reduction is longitudinally localized to the region where the current wedge has been observed to form and that the reduction should develop slowly at first and then more rapidly, as required if plasma sheet divergence driven by diamagnetic drift leads to current wedge formation and thus the initiation of the substorm expansion phase. We also find that the reduction in flux tube content must occur within the plasma sheet mapping of the Harang discontinuity, consistent with observations showing that auroral brightening at substorm onset occurs within the region of the Harang discontinuity.

2. On the Cause of Flux Tube Content Reduction

[9] The observed reduction in flux tube content within the current wedge, reported in Paper 1, could result from either a change to a source population for the inner plasma sheet that has significantly less flux tube content than the source population prior to onset or from significantly enhanced loss of particles from flux tubes of the inner plasma sheet. First, we consider the possibility that the source population changes to one of significant lower flux tube content. Prior to onset, particles drift to the inner plasma sheet by a combination of sunward electric drift and azimuthal magnetic drift. Since magnetic drift is energy dependent, the population within the inner plasma sheet represents a mixture of particles from different source regions in the distance plasma sheet (see *Wang et al.* [2003], and references therein, for discussion of this mixing). During the substorm expansion phase, particles within the current wedge also move sunward in response to the induced electric field associated with the dipolarizing magnetic field. Based on the x-component of ion velocity perpendicular to **B** in Figures 2 and 7 of Paper 1 and in the work of *Lui et al.* [1998], *Miyashita et al.* [2000], and *Ohtani et al.* [2002], the motion perpendicular to **B** during the substorm expansion phase can be estimated to have a maximum of ~300 km/s for ~3 min. This gives a maximum earthward displacement within the equatorial plane of ~10 R_E.

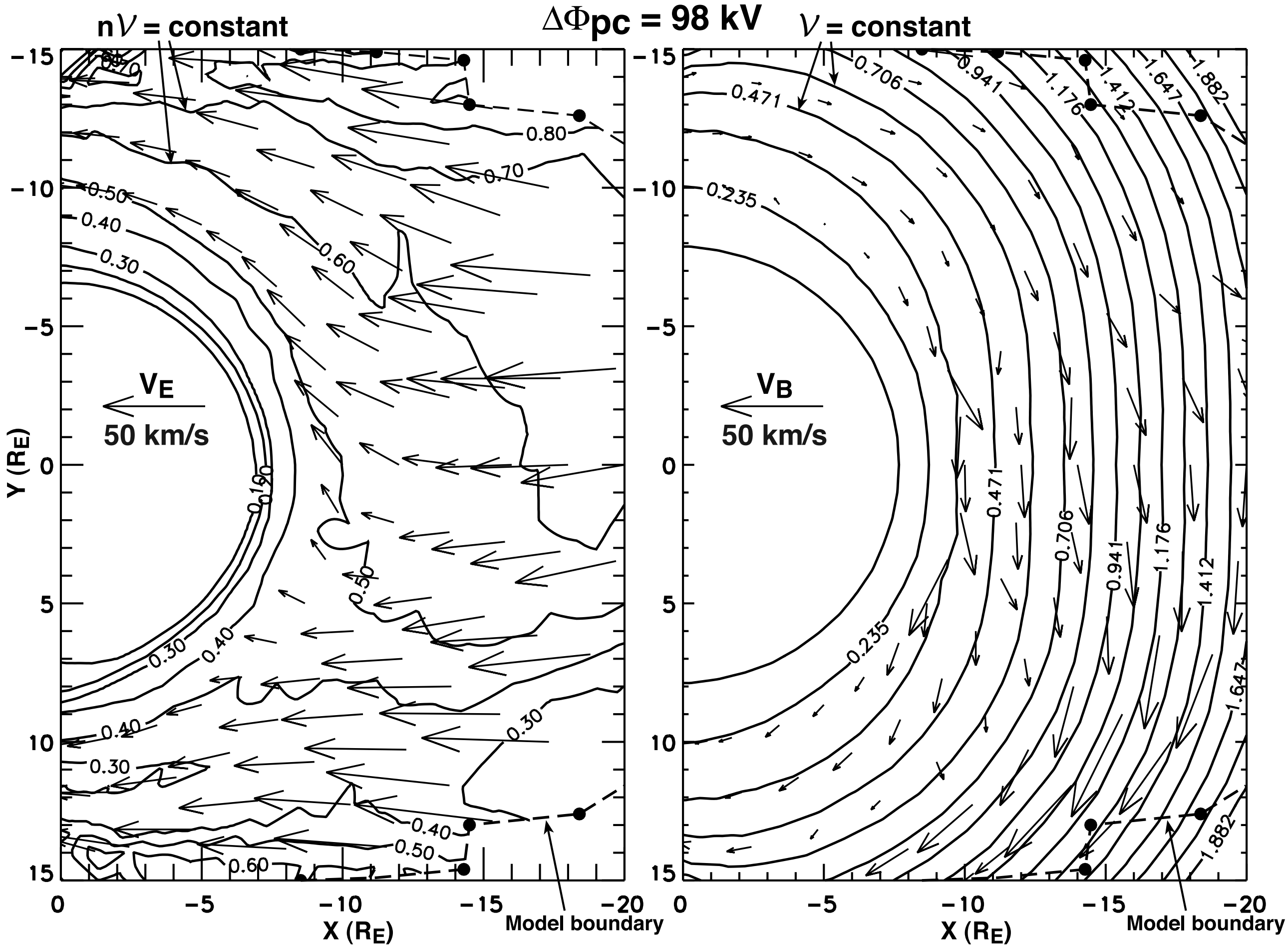

Figure 1. Steady state results in the equatorial plane from the *Wang et al.* [2003] model for a cross polar cap potential drop $\Delta\Phi_{pc}$ of 98 kV. Flux tube volume per unit magnetic flux (in units of R_E/nT) and diamagnetic drift are shown in the right panel. Flux tube particle content (in units of R_E/nT-cm^3) and electric drift are shown in the left panel.

[10] Thus for a reduction in source particles to be responsible for the observed reduction in flux tube content within the inner plasma sheet, there would have to be a region tailward of, but within $\sim$10 R_E of, the observing location having significantly reduced flux tube content prior to substorm onset. *Wang et al.* [2003] have modeled the particle distributions and magnetic fields throughout the $x > -20$ R_E portion of the nightside plasma sheet under the enhanced convection expected during the growth phase of substorms. This model has been constructed to give agreement with plasma moment measurements throughout the equatorial plane within the plasma sheet, and the moments obtained by the model agree quite well with plasma sheet observations. The Wang et al. model maintains force balance within the midnight meridian plane but not throughout the equatorial plane so that the model should not expected to give precisely correct plasma and magnetic field variations of parameters within the equatorial plane. While this is a limitation, the Wang et al. model is the only model of the inner plasma sheet of which we are aware that has been tested with plasma sheet observations and includes magnetic as well as electric drift. As discussed below, magnetic drift is critical to substorm dynamics within the inner plasma sheet. We thus use the results of this model to give an indication of the variations of proton moments and magnetic field within the equatorial plane of the plasma sheet.

[11] The *Wang et al.* [2003] model gives flux tube content as a function of X and Y in the equatorial plane as shown by contours in the left panel of Figure 1. These contours indicate that within the $x > -20$ R_E portion region of the plasma sheet, flux tube content generally increases with distance down the tail. In particular, the contours do not show any evidence for a region with significantly reduced flux tube content that could be transported $\lesssim$10 R_E earthward during the expansion phase and yield the reduction in flux tube content that we have observed near $x = -10$ R_E. It thus is highly unlikely that a change to a source population with reduced flux tube content could account for the expansion phase reduction of flux tube content within the inner plasma sheet.

[12] The observed reduction of flux tube content must therefore be due to particle loss from inner plasma sheet flux tubes during the expansion phase. Proton loss from precipitation to the atmosphere can be neglected, since strong pitch angle diffusion loss rates are several hours or

more for within the plasma sheet [*Kennel*, 1969]. To evaluate other causes for changes in flux tube content, we use the single species mass conservation equation for flux tubes of isotropic particles that was derived by *Heinemann* [1999]. The assumption of isotropic pressure is generally a good assumption for the plasma sheet [*Stiles et al.*, 1978; *Nakamura et al.*, 1991]. *Heinemann* [1999] used the slow flow and slow time variation approximation of *Wolf* [1983], which simplifies the single species momentum equation to

$$\mathbf{V}_{\perp} = \mathbf{V}_E + \mathbf{V}_B. \quad (1)$$

Here $\mathbf{V}_{\perp}$ is the component of velocity perpendicular to **B**, and $\mathbf{V}_E$ and $\mathbf{V}_B$ are the electric and diamagnetic drifts, which are given by:

$$\mathbf{V}_E = \frac{\mathbf{E} \times \mathbf{B}}{\mathbf{B}^2}$$

$$\mathbf{V}_B = \frac{\mathbf{B} \times \nabla \mathbf{P}}{\mathbf{qnB}^2},$$

where q is the particle charge. Using equation (1) and integrating the single species mass conservation equation over the volume of a flux tube, *Heinemann* [1999] obtained the equatorial continuity equation for flux tubes:

$$\left(\frac{\partial}{\partial t} + \mathbf{V}_E \cdot \nabla\right)(n\mathcal{V}) = -n\mathbf{V}_B \cdot \nabla\mathcal{V}, \quad (2)$$

where losses due to precipitation have been neglected.

[13] Equation (2) states that the flux tube content in the frame of reference of the electric field drift changes as a result of particle divergence that results from diamagnetic drift in the direction of the gradient of flux tube volume. This divergence results from magnetic drift, since the particle divergence due to magnetization drift ($\nabla \cdot n\boldsymbol{V_m}$, where the magnetization drift $\boldsymbol{V_m} = -(1/ne)\nabla \times (P_{\perp}\mathbf{B}/B^2)$) is identically zero. Flux tube content is conserved in the frame of reference of the electric drift only if $\mathbf{V}_B \cdot \nabla\mathcal{V} = 0$, which generally does not occur. Steady sate requires that $\mathbf{V}_E \cdot \nabla(n\mathcal{V}) = -n\mathbf{V}_B \cdot \nabla\mathcal{V}$, which expresses balance between particle transfer by electric drift and particle divergence by diamagnetic drift.

[14] During quite times, when the convection electric field is small and plasma pressure is low within the inner plasma sheet, steady state balance can be achieved with low values for $\mathbf{V}_E$ and $\mathbf{V}_D$. The deflection of particles by diamagnetic drift during quiet times gives a large decrease of $n\mathcal{V}$ with decreasing radial distance within the plasma sheet [*Wang et al.*, 2001]. (Wang et al. show $P\mathcal{V}^{5/3}$, but this is proportional to $n\mathcal{V}$ for adiabatic heating). An increase in the convection electric field throughout the plasma sheet, as is expected to occur at the initiation of the substorm growth phase, will increase $|\mathbf{V}_E|$ and thus $|\mathbf{V}_E \cdot \nabla(n\mathcal{V})|$. Since $\mathbf{V}_E \cdot \nabla(n\mathcal{V})$ is negative, we see from equation (2) that an increase in $|\mathbf{V}_E|$ will cause $n\mathcal{V}$ to increase throughout the plasma sheet. This is likely the cause of the increase in plasma pressure within the plasma sheet that occurs during the growth phase of substorms and would be expected to increase $n\mathbf{V}_B \cdot \nabla\mathcal{V}$ and possibly decrease $\nabla(n\mathcal{V})$. These changes should continue until either $\mathbf{V}_E \cdot \nabla(n\mathcal{V}) = -n\mathbf{V}_B \cdot \nabla\mathcal{V}$, in which case steady state would be reached, or until the strength of the convection electric field reduces sufficiently to give $|\mathbf{V}_E \cdot \nabla(n\mathcal{V})| < |n\mathbf{V}_B \cdot \nabla\mathcal{V}|$.

[15] $\mathbf{V}_E$ and $\mathbf{V}_B$ as obtained within the equatorial plane by *Wang et al.* [2003] for steady state conditions of enhanced convection are shown by arrows in the left and right panels of Figure 1, respectively. It can be seen that electric drift brings plasma towards the Earth and diamagnetic drift cause azimuthal deflection around the Earth. It can also be seen that $\mathbf{V}_E$ and $\mathbf{V}_B$ are of the same magnitude, so that both drifts in equation (1) are of approximately equal importance. The magnitude of $\mathbf{V}_E$ and $\mathbf{V}_B$ are also enhanced over their quiet time values.

[16] The particle divergence given by $n\mathbf{V}_B \cdot \nabla\mathcal{V}$ also corresponds to a divergence in cross-tail current, which is related to field-aligned currents via the relation [*Vasyliunas*, 1970; *Birmingham*, 1992]

$$j_{\|,i} = (B_i/B)\hat{\mathbf{B}} \cdot \nabla P_{eq} \times \nabla\mathcal{V} = qB_i(n\mathbf{V}_B \cdot \nabla\mathcal{V}). \quad (3)$$

Here $j_{\|,i}$ is the upward field-aligned current density at the ionosphere, and B and B_i are the magnetic field intensities at the equator and in the ionosphere, respectively. The right-hand panel of Figure 1 shows steady state flux tube volume contours as well as $\mathbf{V}_B$ as obtained by *Wang et al.* [2003]. It is the component of $\mathbf{V}_B$ in the direction of $\nabla\mathcal{V}$ that gives particle divergence, and thus will give upward field-aligned currents as given by equation (3).

[17] Figure 1 indicates that the largest components of $\mathbf{V}_B$ in the direction of $\nabla\mathcal{V}$, and thus the largest upward field-aligned currents, should occur at $X \sim 10-18\ R_E$ within the midnight to premidnight region. This will lead to upward field-aligned currents, which must be balanced by converging ionospheric currents. Neglecting ionospheric conductivity gradients, it is necessary that converging electric fields develop within the ionosphere to provide the necessary converging currents. The converging electric fields formed in this way give rise to the reversal from equatorward directed to poleward directed ionospheric electric fields known as the Harang discontinuity, as described in detail by *Erickson et al.* [1991]. Equation (3) implies that enhancement of plasma pressure within the inner plasma sheet, as occurs during the growth phase of substorms, should lead to intensification of the Harang discontinuity, and such intensification has been observed by *Bristow et al.* [2001].

[18] It can be seen from equation (2) that a reduction in convection that causes $|\mathbf{V}_E \cdot \nabla(n\mathcal{V})| < |n\mathbf{V}_B \cdot \nabla\mathcal{V}|$ will lead to a divergence of plasmas sheet particles that gives $(\partial/\partial t)(n\mathcal{V}) < 0$. This will lead to a reduction in $n\mathcal{V}$, as we have found to occur during the expansion phase of substorms. It thus offers a plausible explanation for why substorm onset is generally caused by a reduction in the strength of the large-scale convection that is imparted to the magnetosphere from the solar wind. Also, the same particle divergence gives rises to both the Harang discontinuity during the substorm growth phase (equation (3)) and to the reduction in $n\mathcal{V}$ when the strength of convection is reduced (equation (2)). Thus following a reduction in $\mathbf{V}_E$, maximum rates of reduction of $n\mathcal{V}$ should occur

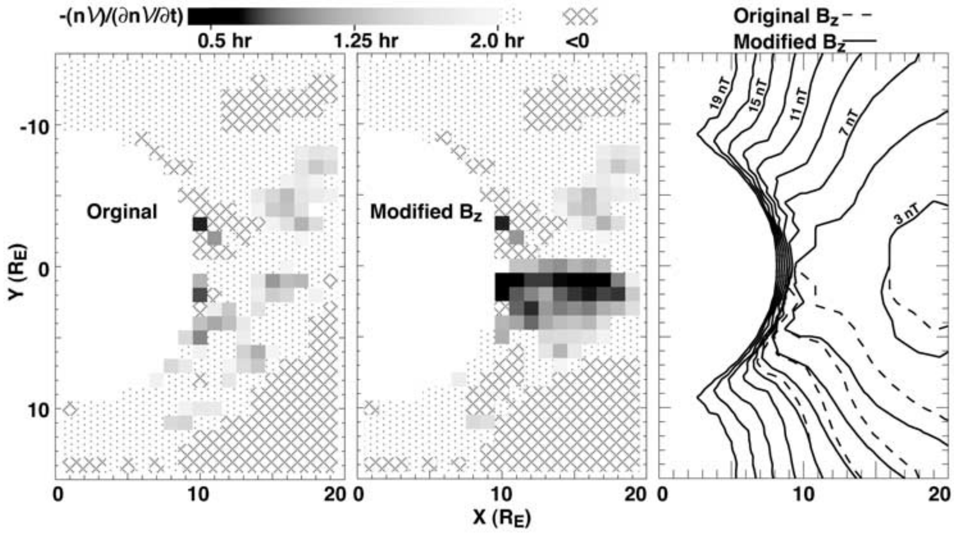

Figure 2. $(\mathbf{V}_D \cdot \nabla\mathcal{V}/\mathcal{V})^{-1}$ as obtained directly from the *Wang et al.* [2003] steady state results $\Delta\Phi_{pc}$ of 98 kV (left panel) and as obtained from the same model with B_z artificially modified (middle panel) as shown by contours in the right-hand panel.

within the region of the equatorial mapping of the Harang discontinuity.

[19] If a reduction in the strength of convection leads to the substorm expansion phase via the divergence given by $-n\mathbf{V}_B \cdot \nabla\mathcal{V}$, such divergence should initially lead to relatively slow growth of the substorm current wedge and then lead to more rapid, nonlinear growth. Equation (2) allows us to estimate how fast the particle divergence will deplete flux tubes of particles after a reduction in $|\mathbf{V}_E|$. For simplification, let us assume that $|\mathbf{V}_E|$ is reduced to zero. Then equation (2) gives the time scale for a reduction in $n\mathcal{V}$ as

$$\tau = -n\mathcal{V}/\left(\frac{\partial}{\partial t}n\mathcal{V}\right) = (\mathbf{V}_B \cdot \nabla\mathcal{V}/\mathcal{V})^{-1}. \quad (4)$$

An accurate evaluation of the right-hand side of equation (4) would require the forced balanced distribution of $\mathbf{V}_B$ and $\mathcal{V}$ throughout the equatorial plane, which is not available. We therefore turn to the model of *Wang et al.* [2003].

[20] Figure 2 shows two different estimates for τ. The left hand panel shows $(\mathbf{V}_B \cdot \nabla\mathcal{V}/\mathcal{V})^{-1}$ as obtained directly from the *Wang et al.* [2003] steady state results shown in Figure 1. This is clearly an underestimate in the post-midnight region, because the model flux tube volume for a fixed radial distance decreases away from midnight in both the duskward and dawnward direction. Such a decrease, however, is not realistic in the duskward direction because the increase in plasma pressure that occurs in that direction would be expected to further inflate the magnetic field. However, this inflation is not included in the model because self-consistency with the plasma is not maintained off the midnight meridian plane. Despite this, the model shows a region that is localized between Y = 1 and 5 R_E and X = 10–18 R_E where τ is on the order of 1 hour. This would decrease $n\mathcal{V}$ in the correct region for current wedge formation and give an ~8% decrease over a 5 min slow-growth phase for the current wedge.

[21] As a first attempt to estimate the effects of magnetic field inflation toward the dusk side, we have artificially reduced B_z in the *Wang et al.* [2003] model at 20.7 to 24 MLT as indicated by the contours in the right hand panel of Figure 2. (Specifically, a ΔB_z was added between $r = 9$ and 25 R_E and between $\phi = 130°$ to $180°$ (20.7 to 24 MLT) given by $\Delta B_z/B_z = -\cos[(\phi - 155°)/25°][0.3 - 0.2(r - 9R_E)/16R_E])$. With this adjustment, the time scale for reduction in $n\mathcal{V}$ reduces to as low as ~30 min in the premidnight region, giving an ~15% decrease over a five minute slow-growth phase.

[22] The above estimates are quite rough. However, as indicated by equations (2) and (3), the existence and location of the Harang discontinuity during the substorm growth phase tell us that our evaluation of the divergence term is at least qualitatively correct. The evaluation indicates that slow growth of the current wedge formation should occur in the premidnight to midnight region of the near-Earth plasma sheet, which is the region where the current wedge has been observed to initially form [e.g., *Nagai*, 1991], and within the region of the Harang discontinuity in the ionosphere as observed. However, the above time scales are far too long to account for the rapid current wedge growth that occurs during the substorm expansion phase.

[23] For plasma sheet divergence to be responsible for the substorm expansion phase, it must lead to nonlinear growth of the current wedge, i.e., the rate of growth of the current wedge must increase as the current wedge grows. An appropriate self-consistent model, without use of the slow-flow approximation used to derive equation (2), would be necessary to quantitatively evaluate such nonlinearity. However, a qualitative look at the pressure and flux tube volume changes associated with current wedge formation indicates that the nonlinearity is a probable result of the longitudinally localization of the current wedge.

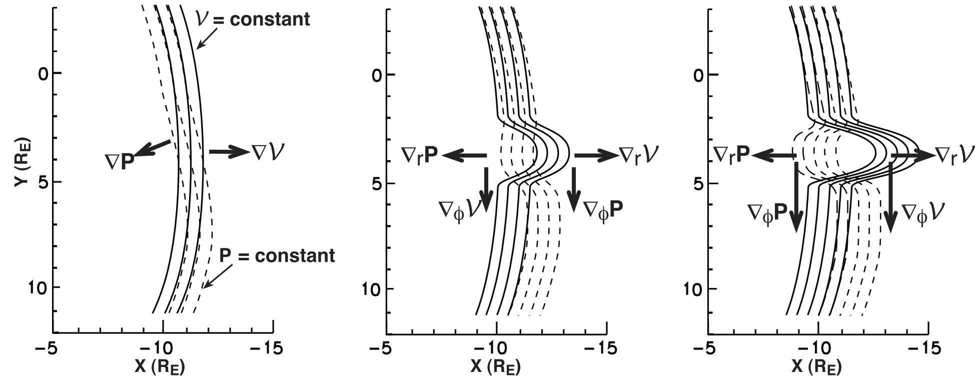

Figure 3. Illustration of contours of constant P_{eq} and constant $\mathcal{V}$ in the equatorial plane prior to expansion phase onset (left panel) and during the growth of the substorm current wedge (middle and right panels). Illustration is in rectangular coordinates, but X and Y should be thought of as being in the radial and azimuthal directions, respectively.

[24] To show this, we rewrite equation (4) as the rate of reduction of flux tube content $\gamma = \tau^1$ as:

$$\gamma = \frac{\nabla P_{eq} \times \nabla \mathcal{V} \cdot \hat{\mathbf{B}}_z}{qn\mathcal{V}B_z} = \left(\nabla_\phi P_{eq} \nabla_r \mathcal{V} - \nabla_r P_{eq} \nabla_\phi \mathcal{V}\right)/(qn\mathcal{V}B_z), \quad (5)$$

where ∇_r and ∇_φ are the radially outward and westward components of ∇, respectively. Since $\mathcal{V} \propto B_z^{-3/2}$ to B_z^{-1} (see Paper 1), and B_z increases and n decreases during the expansion phase, the denominator of equation (5) will decrease in association with current wedge formation. This will increase γ with time as the current wedge forms. Also, both $|\nabla_\varphi \mathcal{V}|$ and $|\nabla_\varphi P_{eq}|$ should increase significantly as the current wedge forms, as illustrated in Figure 3, which will also cause an increase of γ with time as the current wedge forms. Figure 3 schematically shows contours of constant P_{eq} and constant $\mathcal{V}$. Diamagnetic drift is directed along contours of constant P_{eq}. Figure 3 is drawn in rectangular coordinates for simplicity, but X and Y should be thought of as being in the radial and azimuthal directions, respectively.

[25] Current wedge formation is expected to initiate in the region where the contours of constant P_{eq} and $\mathcal{V}$ are not parallel, as illustrated in the left-hand panel of Figure 3, giving $\nabla \mathcal{V} \times \nabla P_{eq} \neq 0$. This gives a slow, longitudinally localized reduction in P and in the cross-tail current. Reduction in cross-tail current gives dipolarization of the magnetic field, which is accompanied by an increase in B_z and decrease in $\mathcal{V}$. The decrease in P_{eq} and in $\mathcal{V}$ will distort the contours of constant P_{eq} and $\mathcal{V}$ as illustrated in the middle and right-hand panels of Figure 3. This distortion of the contours is qualitatively as expected to occur as the current wedge forms. However, such a distortion leads to azimuthal gradients in P_{eq} and $\mathcal{V}$ as illustrated in the middle panel of Figure 3, and these azimuthal gradients will grow as the current wedge strengthens as illustrated in the right-hand panel. As can be seen from equation (5), these azimuthal gradients will increase γ on the duskward side of the current wedge. The growth rate γ will continue to increase as the current wedge strengthens provided there are no equivalent reductions in the magnitude of the radial gradients of P_{eq} and $\mathcal{V}$, and we know of no reason why such a significant reduction should occur. As a simple estimate, a factor of 2 decrease in n, coupled with a factor of 2–3 increase in the contributions from each of the two terms in the numerator of equation (5), would give an order of magnitude increase in γ, which would readily account for the rapid current wedge formation that occurs during the expansion phase of substorms. Equations (4) and (5) apply in the frame of reference of the dipolarizing magnetic field, so that the above arguments include the effects of the electric field induced as the magnetic field dipolarizes.

[26] We thus see that plasma sheet divergence following a reduction in the strength of large scale convection should lead to a growth rate for the current wedge that will increase as the current wedge strengthens and thus offers a plausible explanation for the initial slow growth and transition to rapid growth that characterizes current wedge formation during the expansion phase of substorms.

3. Summary and Conclusions

[27] In Paper 1, we presented Geotail spacecraft measurements obtained near the center of the plasma sheet at r ~ 10–13 R_E and found that a reduction in equatorial proton pressure generally occurs within the substorm current wedge. While temperature increases in association with the dipolarization of the magnetic field, the plasma density decreases as the current wedge forms and is large enough to lead to a decrease in pressure. Combining the decrease in density with the decrease in flux tube volume that also occurs as the magnetic field dipolarizes shows that there is a large reduction in flux tube ion content within the current wedge. Understanding the cause of this ion loss is thus necessary for understanding the substorm expansion phase.

[28] To investigate the cause of the flux tube ion loss, we have in this paper applied the continuity equation for plasma sheet particles derived by *Heinemann* [1999]. We found that an enhancement in convection should lead to an enhancement of the flux tube content within the inner plasma sheet, as is expected to occur during the substorm growth phase. It should also lead to an enhancement of the strength of the Harang discontinuity during the growth phase, as has been observed [*Bristow et al.*, 2001]. A reduction in the strength of convection following a period of enhanced convection

was found to cause a divergence of plasma sheet particles that is driven by magnetic drift. This divergence was found to give a slow reduction in flux tube content within the premidnight to midnight region and could thus account for the slow growth of the current wedge that has been inferred to occur during the few minutes prior to substorm onset. This process for current wedge formation must initiate within the equatorial mapping of the Harang discontinuity, since the same plasma sheet particle divergence drives both the Harang discontinuity during the growth phase and the flux tube particle loss following a reduction in the strength of convection.

[29] The time scale for the slow development of the current wedge was found to be far too long to account for the rapid current wedge growth that occurs during the substorm expansion phase. However, we also found that the density, pressure, and flux tube volume reduction that occurs within the current wedge should increase that rate of particle divergence thus causing the growth rate of the current wedge to increase as the current wedge strengthens. This would lead to non-linear enhancement of current wedge growth, which could account for the rapid current wedge formation that occurs during the expansion phase of substorms.

[30] Our theoretical analysis has by necessity been quite qualitative. To quantitatively evaluate our proposal for the nonlinear evolution of current wedge formation during the substorm expansion phase would require an appropriate model. Such a model must include a time dependent magnetic field that maintains three-dimensional self-consistently with the plasma. The model should include the acceleration term in the momentum equation, which is not included in the *Heinemann* [1999] continuity equation, in order to address rapid changes during the expansion phase. Also, the model must not make the assumption $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$ of ideal MHD. This assumption is equivalent to setting $\mathbf{V}_B = 0$ in equation (1), which is clearly inappropriate in the inner plasma sheet and would cause the divergence term on the right-hand side of equation (2) to be zero. Additionally, $\nabla \cdot \mathbf{q}$ should not be required to be zero in the energy equation, where $\mathbf{q}$ is the heat flux vector. This is because $\nabla \cdot \mathbf{q}$ involves the energy-depend magnetic drift, as does the right-hand side of equation (2), so that setting $\nabla \cdot \mathbf{q} = 0$ is equivalent to setting the right-hand side of equation (2) equal to zero [*Heinemann*, 1999]. Ideally, effects of coupling to the ionosphere should be included. It would also be interesting to consider the tailward and azimuthal expansion of the current wedge, which could be related, respectively, to the tailward propagation of the reduction in the strength of convection and the westward magnetic drift of plasma sheet protons [*Lyons*, 1995].

[31] The plasma sheet continuity equation (2) offers clear explanations for variations in the strength of substorms, for variations in the speed of development of substorms, for multionset substorms, and for pseudo-breakups. The strength of a substorm should depend on the magnitude of the divergence term on the right-hand side of equation (2) relative to that of $\mathbf{V}_E \cdot \nabla(n\mathcal{V})$, which should depend on the strength and duration of the enhanced convection prior to onset and on the amount of reduction in the strength of convection associated with the onset. The speed of development of a particular expansion would be expected to depend on the time scale over which the strength of convection is reduced, a slow reduction in convection from a slow IMF change giving a slower development than a more rapid reduction in convection. A reduction in convection that occurs in more than one discrete step would yield a substorm with an expansion onset associated with each step, thus accounting for multi-onset substorms. A convection reduction that is soon (in $\lesssim$5–10 min) followed by a convection enhancement would be expected to result in a pseudo breakup. This would lead to normal initiation of a substorm, but a return to growth phase conditions would occur prior to full development of the current wedge.

[32] Our results predict that within the region where the current wedge initially forms, particle fluxes, flux tube particle content, and plasma pressure should all begin to slowly decrease a few minutes prior to ground onset. The observations shown in the companion paper show evidence that low-energy ion fluxes begin to decrease a few minutes prior to ground onset, whereas the plasma moments do not show a consistent change prior to onset. A determination of when these changes initiate within the equatorial inner plasma sheet relative to the time of ground onset would provide an important test for the proposal put forth here. However, motion of the spacecraft relative to the equatorial plane and turbulence associated with the onsets preclude us from reliably making such a determination with the observations in the companion paper.

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